Multi-layer Flight Control Synthesis and Analysis of a Small-scale UAV Helicopter

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***Abstract*—In this paper, we present a systematic procedure for the design of a controller for an autonomous helicopter. The studied model of the helicopter is a semi-linearized model, which includes some nonlinear parts that are separated from the linear blocks. The controller structure is a hierarchical controller which consists of two layers: *the inner-loop controller* that covers the linear parts of the model and *the outer-loop controller* that handles the nonlinear parts. The inner-loop controller aims at the attitude control of the helicopter and the outer-loop is responsible for its position control. The performance of the designed controller is demonstrated through the simulation and actual flight tests in the hovering situation and the path-tracking mode.**

***Index Terms*—Unmanned Aerial Vehicles, Hierarchical Control, Inner-loop Controller, Outer-loop Controller.**

I. INTRODUCTION

Unmanned Aerial Vehicles (UAVs) have attracted the attention from the academic and military communities due to their various capabilities and their potential to address numerous applications in civilian and military areas (see e.g., [1], [2], [3], [4]). An essential issue in the aerial robotics is designing a reliable controller for the UAVs to make them able to perform flight missions autonomously. However, due to the complex structure of the UAVs, their controller design is not an easy task and has motivated several research groups to focus on the design and improvement of their UAV controller performance [1], [5], [6]. In this paper, we propose a systematic controller design procedure for the UAV system. Our test bed is a helicopter which is developed by our research group in the National University of Singapore [7], [8]. The proposed controller has a hierarchical structure, which consists of two layers: *The inner-loop controller* and *The outer-loop controller*. The inner-loop is responsible for the control of the attitude of the UAV and the outer-loop is used to drive the UAV into the desired position. The model of this UAV is a semi-linearized model, in which the nonlinear terms are separated from the linear parts and are brought into the outer-loop. Therefore, the inner-loop includes only the linear parts and we use the H_∞ control design technique to achieve the desired performance and to reduce the disturbance effect. For the outer-loop, we first compensate for the nonlinear terms, and then, we use a proportional controller (P-controller) that enables the system to follow the desired path.

The rest of this paper is organized as follows. In Section II, the structure and the model of the NUS UAV are described. It is a decoupled system which includes two independent subsystems. The controller structure for this model of UAV is explained in Section III. The actual flight tests are conducted to evaluate the controller performance. The experimental results are demonstrated in Section IV. Finally, the paper is concluded in Section V.

II. SEMI-LINEARIZED MODEL OF THE UAV

To construct a UAV helicopter (Fig. 1), one can buy a commercial radio-control helicopter and upgrade it to be able to perform an automatic flight. To develop such an autonomous vehicle, it is necessary to design a reliable avionic system including the following parts:

- Onboard computer system and its extension boards that are responsible for receiving data from the sensors, processing the collected data, and generating the control signals.
- The sensors that are responsible for measuring the state variables such as velocity, acceleration, fuel level, attitude, and etc.
- The servos that are used to manipulate the angles of the helicopter blades to move the UAV towards the desired direction.

In [9], the procedure of the construction of such a UAV helicopter is described, and in [7], its hardware parts including both the avionic system and the ground station are illustrated in detail.

The model of the UAV can be obtained based on basic physical principles. However, the achieved model might be a nonlinear model [8]. As a rational way, we have linearized the UAV model at the hovering state, in which the linear and angular velocities, the pitch angle, and the roll angle are kept close to zero. The resulting semi-linearized model of the NUS

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Fig. 1. NUS UAV helicopter.

UAV has been presented in [6]. The model consists of two decoupled subsystems as follows:

$$\begin{cases} \dot{x}_1 & = A_1 x_1 + B_1 u_1 \\ \dot{x}_{out_1} & = C_1 x_1 \end{cases} \tag{1}$$

and

$$\begin{cases} \dot{x}_2 & = A_2 x_2 + B_2 u_2 \\ \dot{x}_{out_2} & = R' C_2 x_2 \end{cases} \tag{2}$$

where $x_1 = [\ V_z(m/s)\ \omega_z(rad/s)\ w_{zf}(rad/s)\ ]'$ and $x_2 = [\ V_x(m/s)\ V_y(m/s)\ \omega_x(rad/s)\ \omega_y(rad/s)\ \phi(rad)\ \theta(rad)\ \tilde{a}_1(rad)\ \tilde{b}_1(rad)\ ]'$ are the inner-loop state variables; $x_{out_1} = [\ z(m)\ \psi(rad)\ ]'$ and $x_{out_2} = [\ x(m)\ y(m)\ ]'$ are the outer-loop state variables; $u_1 = \left[\ \delta_{col}(rad) \quad \delta_{pedal}(rad)\ \right]'$, and $u_2 = \left[\ \delta_{roll}(rad) \quad \delta_{pitch}(rad)\ \right]'$ are the control inputs. Here, V_x, V_y, and V_z are the linear velocities; ω_x, ω_y, and ω_z are the angular velocities; ϕ, θ, and ψ are the Euler angles; $\tilde{a}_1$ and $\tilde{b}_1$ are the flapping angles; and w_{zf} is the state variable of the rate gyro used to describe the first order differential equation of δ_{pedal} [10]; δ_{roll} and δ_{pitch} are the cyclic commands, and δ_{pedal} is the pedal channel input that affects the yaw angle, and δ_{col} is the collective channel. The saturation level for all of the control inputs is ± 0.5.

Matrices A_1, A_2, B_1, B_2, C_1, C_2, and R are as follows:

$$A_1 = \begin{bmatrix} -0.6821 & -0.1070 & 0 \\ -0.1446 & -5.5561 & -36.6740 \\ 0 & 2.7492 & -11.1120 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 15.6491 & 0 \\ 1.6349 & -58.4053 \\ 0 & 0 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix},$$

$$A2 = \left[\begin{matrix} -0.1778 & 0 & 0 & 0 & 0 & -9.7807 \\ 0 & -0.3104 & 0 & 0 & 9.7807 & 0 \\ -0.3326 & -0.5353 & 0 & 0 & 0 & 0 \\ 0.1903 & -0.2940 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \end{matrix}\right.$$

$$\left.\begin{matrix} -9.7808 & 0 \\ 0 & 9.7807 \\ 75.7640 & 343.86 \\ 172.620 & -59.958 \\ 0 & 0 \\ 0 & 0 \\ -8.1222 & 4.6535 \\ -0.0921 & -8.1222 \end{matrix}\right]$$

$$B_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0.0496 & 2.6224 \\ 2.4928 & 0.1740 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

and

$$R = \begin{bmatrix} cos\,\psi & -sin\,\psi \\ sin\,\psi & cos\,\psi \end{bmatrix}.$$

The model structure of the UAV is shown in Fig. 2. Subsystem 1 and Subsystem 2 are similar; however, in Subsystem 2, there exists the term R' as a nonlinear term that makes the design procedure different. In the following section, the design control procedures for this model of UAV is discussed in detail.

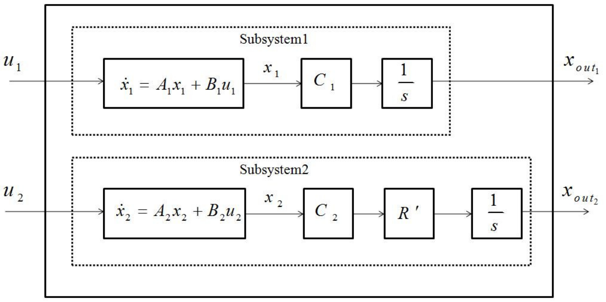

Fig. 2. The model structure of the UAV.

III. CONTROLLER DESIGN

We design the flight controller in two steps. First, we control the attitude of the UAV in the inner-loop, and then, we will control the position of the UAV and drive it towards the desired position in the outer-loop. The inner-loops of both subsystems are linear and therefore, we can use the H_∞ control design technique to stabilize the system and achieve the desired performance. Also the designed controller will reduce the disturbance effect on the control performance. For the outer-loop, we use a P-controller to track the position references. The design procedure for both subsystems is similar. However, Subsystem 2 has the nonlinear term R', in its outer-loop that makes the design procedure more difficult. We need to first compensate for this nonlinearity, and then, design a P-controller for the system to achieve the required performance. In the following parts, we will describe the controller design procedure for the inner-loop and the outer-loop of the system.

A. Inner-loop Controller Design

The inner-loop controllers in both subsystems aim at the control of the attitude of the UAV. The attitude of the UAV is captured by the state variables x_1 and x_2 and their corresponding linear dynamics, as described in (1) and (2). For these linear multi-variable systems, we can use any of the classical control tools. However, with the H_∞ control design technique, both the robust stability and the desired performance of the system are achievable [11]. Utilizing the H_∞ control design technique, we have designed the control law in the form of $u_i = F_i x_i + G_i r_i$, $i = 1, 2$, where F_i can be obtained from the H_∞ design technique, and G_i, $i = 1, 2$, are the feedforward gains that can be calculated as $G_i = -(C_i(A_i + B_iF_i)^{-1}B_i)^{-1}$. Using this method, the matrices F_1, G_1, F_2, and G_2 are as follows:

$$F_1 = \begin{bmatrix} -0.0935 & -0.0005 & 0.0027 \\ 0.0008 & 0.0364 & -0.0481 \end{bmatrix}$$

$$G_1 = \begin{bmatrix} 0.1371 & 0.0066 \\ -0.0020 & -0.2748 \end{bmatrix}$$

$$F2 = \left[\begin{matrix} 0.0017 & -0.1683 & -0.0486 & 0.0081 \\ 0.0815 & -0.0461 & -0.0087 & -0.0535 \end{matrix} \right.$$
$$\left. \begin{matrix} -1.9336 & -0.1974 & -0.3227 & -2.1444 \\ -0.3908 & -1.0690 & -1.1712 & -0.4659 \end{matrix} \right]$$

and

$$G_2 = -(C_2(A_2 + B_2F_2)^{-1})^{-1} = \begin{bmatrix} -0.0029 & 0.2335 \\ -0.0978 & 0.0632 \end{bmatrix}.$$

To simulate the resulting controlled system and evaluate the controller performance, we ran a simulation (Fig. 3) with the initial state of $x_1(0) = [1\ 0\ 0]'$ and $x_2(0) = [1\ 0\ 0\ 0\ 0.25\ 0\ 0\ 0]'$. The controlled system reaches the steady hovering state after 3 second, while the control inputs are within the unsaturated region.

B. Outer-loop Controller Design

In the outer-loop of Subsystem 1, we have used a P-controller to drive the UAV to follow the desired path. Before designing the P-controller, we have compensated for the nonlinear term R' in the outer-loop of Subsystem 2. The nonlinearity introduced by R', can be compensated by the term R, as it is a transformation matrix and it has the property that $RR' = I$. To explain the elimination of the nonlinear terms in Subsystem 2, the designed control structure is shown in Fig. 4, where $G_{in_2} = C_2(SI - (A_2 + B_2F_2))^{-1}B_2G_2$ is the inner-loop block. Figure 5 shows that the block G_{in_2} is very close to a decoupled system with equal diagonal elements. Using this property, we can exchange the blocks G_{in_2} and R so that the rotation matrices R and R' will cancel each other.

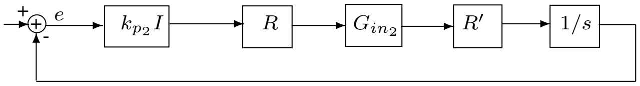

Fig. 4. The control structure of Subsystem 2.

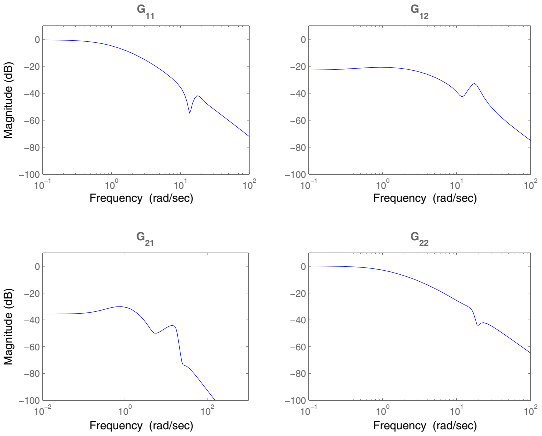

Fig. 5. Bode plot of the entries of G_{in_2}.

After the cancelation of the nonlinear terms, we can design the proportional control gains K_{p_1} and K_{p_2} using generalized Nyquist theorem [12]. The control gains K_{p_1} and K_{p_2} were obtained as follows:

$K_{p_2} = diag\{0.3\,,\ 0.3\}$, $K_{p_1} = diag\{0.5,\ 0.7\}$

In these control parameters, the diagonal elements of K_{p_2} are selected equal, due to the fact that the behavior of the UAV system in the x and y directions should be the same, but for K_{p_1} we do not have this kind of restriction. The whole control structure is shown in Fig. 6.

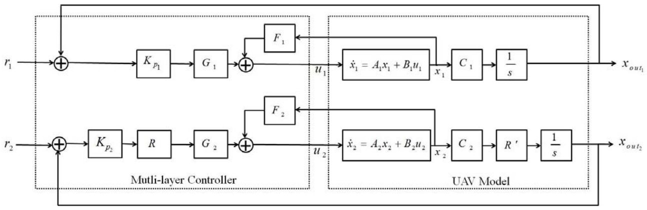

Fig. 6. Multi-layer control structure o the UAV.

To simulate the resulting system, let the outer-loop reference at $r_1 = (1, 0.5)$ and $r_2 = (1, 1)$, and consider that the current position of the UAV is located at $x_{out_{1_0}} = (0, 0)$ and $x_{out_{2_0}} =$

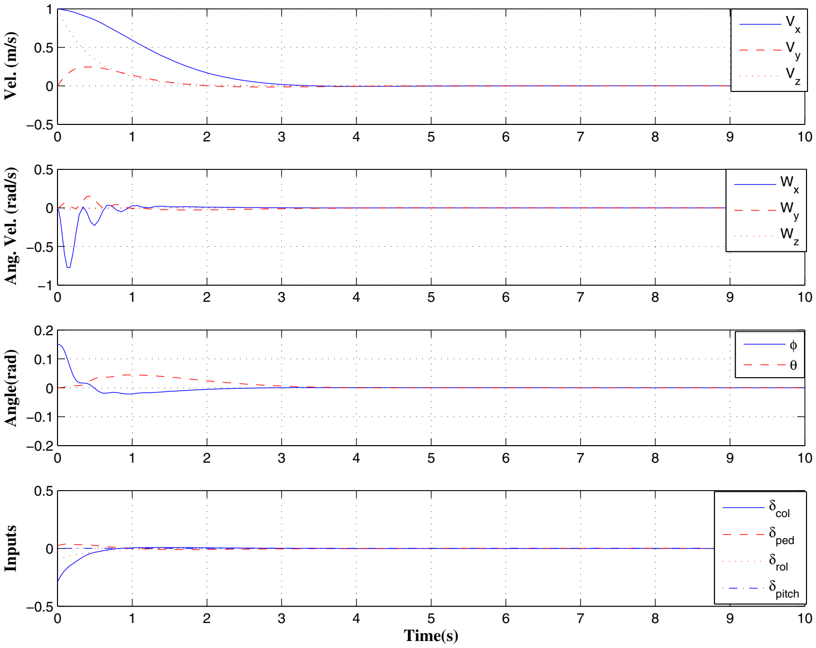

Fig. 3. Simulation of the inner-loop.

$(0, 0)$. The system will smoothly reach its target after around 8 second, (Fig. 7).

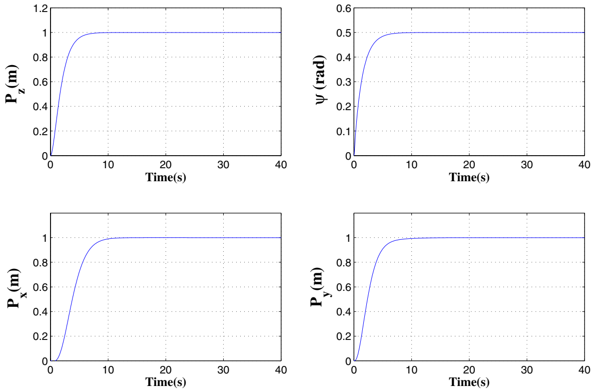

Fig. 7. Simulation of the outer-loop.

IV. EXPERIMENTAL RESULTS

We used the designed controller in an actual flight test to verify the simulation results. First, we used the UAV in a hovering situation at $(x, y, z, \Psi) = (-30, -10.3, 8, -1.3)$ for 15 sec, as shown in Fig. 8. The result is quite impressive, as during this period, the UAV deviates from the desired point at most 1 meter (Fig. 9). The control inputs are shown in Fig. 10. All the control inputs are within the unsaturated region.

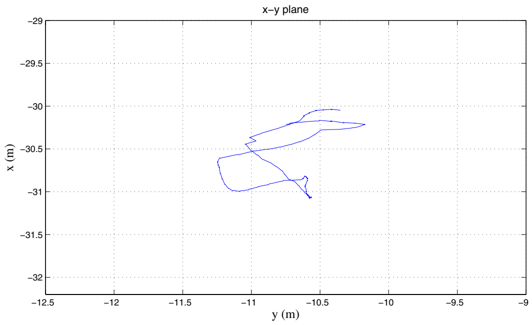

Fig. 9. UAV position in the $x - y$ plane in the hovering.

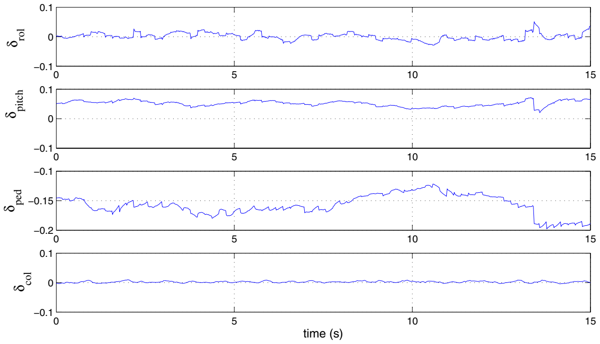

Fig. 10. Control signals in the hovering.

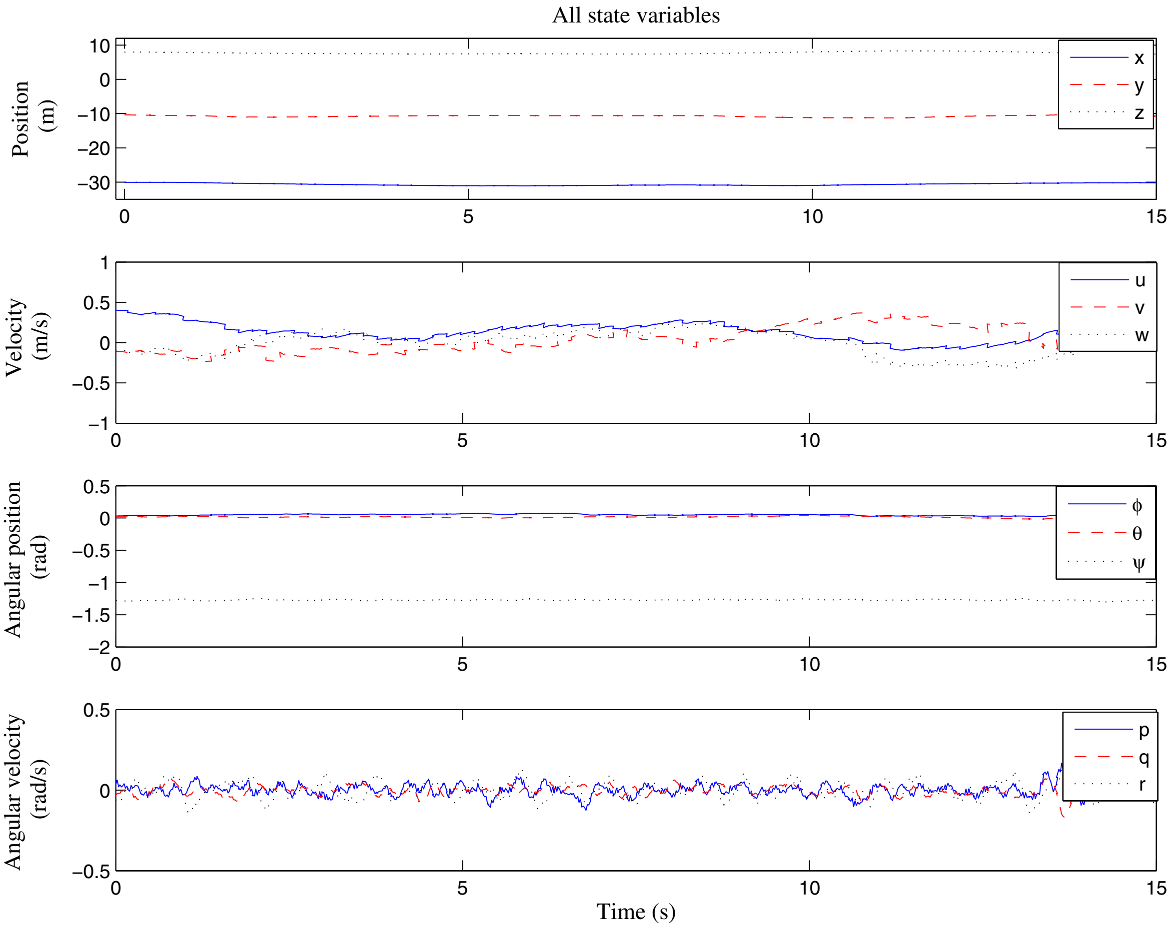

Fig. 8. State variables of the UAV in the hovering.

Next, we have driven the UAV to track a zigzag path as a given trajectory. In this experiment the altitude and the heading angle of the UAV remain unchanged (Fig. 11). To evaluate the controller performance, both the reference path and the UAV position are shown in Fig. 12.

The small errors in these two experiments can be because of the environmental effects such as wind disturbance and the inaccuracy of the GPS sensor. The results show that the UAV is able to track the desired trajectory as long as the generated path does not push the UAV far from the hovering state. The videos of the hovering and path tracking experiments are available at the web links, http://uav.ece.nus.edu.sg/video/hover.mpg and http://uav.ece.nus.edu.sg/video/zigzag.mpg, respectively.

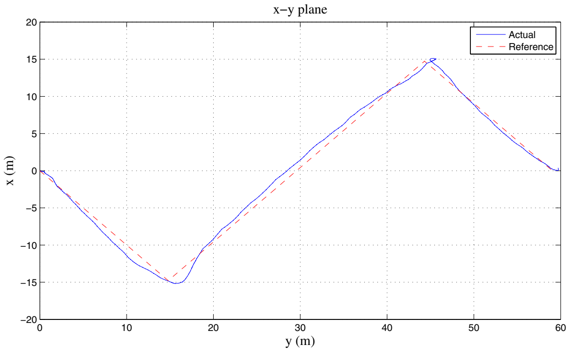

Fig. 11. Tracking a desired path in $x-y$ plane.

V. CONCLUSION

In this paper, we proposed a hierarchical controller for the NUS UAV helicopter. The lower level is an H_∞ controller, which is responsible for the control of the attitude of the UAV. It provided a stable and robust control performance for the UAV in the hovering mode. The higher level aims at the position control of the UAV and consists of a P-controller combined with a nonlinear transformation. This

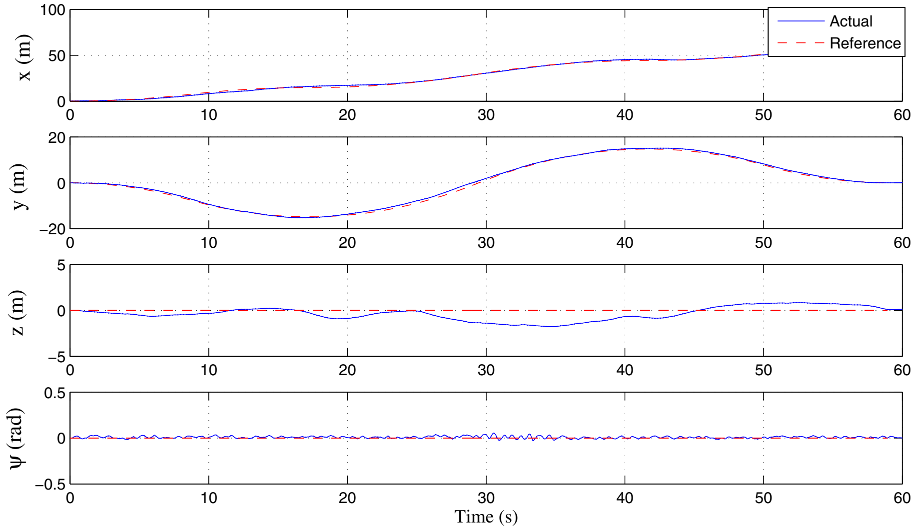

Fig. 12. Tracking the generated path.

control structure makes the UAV able to perform the path tracking mission. The experimental results show that the UAV is able to follow the generated path as long as it respects the physical constraints on the UAV motion. The controlled system was evaluated through the hovering and path tracking experiments. The simulation results and actual flight tests show the effectiveness of the designed controller.

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