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Developments in Hybrid Modeling and Control of Unmanned Aerial Vehicles

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Abstract— **Hybrid modeling and control issues provide a flexible powerful tool for Unmanned Aerial Vehicles (UAVs) as complex control platforms. This paper aims to briefly review recent results in this area. Firstly, existing works on hybrid modeling and control of a single UAV are reviewed. Then multiple UAVs control in a synchronously cooperative task planning, which gives much more maneuverability, has been surveyed in the hybrid framework. Formation control of UAVs is a typical cooperative strategy with considerable applications in multi-agent aerial robotics area and can leverage limited abilities of single UAVs into complicated tasks of group missions. There are a few works in this field which we briefly reviewed their results and explored the possible future directions of these research activities.**

I. INTRODUCTION

Nowadays, Unmanned Aerial Vehicles (UAVs) in many research activities are used as excellent test-beds to implement control schemes due to their complex dynamics and also due to their applications in various military and civilian areas (see, e.g., [1], [2], [3], [4]). For instance in [4] a linearized model of an autonomous helicopter (Fig.1) has been developed and identified to be controlled in the hovering state using composite nonlinear feedback (CNF) controller.

The tasks like Take off, Landing, Hovering, and Cruising are common tasks that a typical UAV is required to accomplish. In most research projects, an especial model has been proposed for each task, and consequently a particular controller has been designed. Therefore, we may have switching between controllers due to maneuver change of the UAV. Such a switching control strategy implies that continuous dynamic of the system is subjected to a discrete dynamic which is mostly neglected in these works. The ignorance of the interaction between continuous and discrete dynamics of the system is questionable and can lead to big failures like the explosion of Ariane5 rocket On June 4, 1996.

A traditional approach to build a supervisory control for the switching logic of the UAV dynamic was hierarchical structure approach (see, e.g., [5]). In this structure, the higher level or outer loop is responsible for the task scheduling and decision making and the lower level or inner loop is responsible for attitude control of the UAV and drive it toward the steady state. This classification of the levels of the controllers is only for simplifying the design and implementation. In the hierarchical approach, these two levels of the controllers will be designed and implemented separately; However, hybrid modeling and control provide a unified framework to consider both discrete and continuous parts of the system, simultaneously. Hybrid modeling also enables the designed controller to be used modularly in a multi-agent scheme. Therefore, supervisory hybrid control approach has aroused a great interest in the academic recent research activities due to its ability to address these challenging problems especially in the field of aerial robotics; However, only a few number of existing works have focused on the control of UAVs in the hybrid framework because of the practical difficulties.

In this paper, the foundation of hybrid modeling and control are reviewed In Section II. Section III is devoted to the existing works on the hybrid modeling of a single UAV. Section IV focuses on the formation and cooperative control of multi-UAV systems. The paper is concluded in Section V.

Fig. 1. Autonomous UAV Helicopter of NUS University.

II. HYBRID MODELING AND CONTROL

Hybrid systems are heterogeneous structures consist of two interactive parts: discrete part and continuous part. This structure provides a mathematical representation and analysis tools for a variety of applications ranging from manufacturing and chemical process to robotics and aerospace control [6], [7], [8]. A useful tool for describing a hybrid system is

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hybrid automaton.
A hybrid automaton is a tuple H = (V, X, F, Init, Inv, Jump) in which:

- V is a finite set of *locations* corresponding to the discrete states of the system.
- X$\subset \Re^n$ is the continuous state space of the system.
- Init$\subset X \times \Re^n$ is the initial state set of the system.
- $F : X \longrightarrow 2^{\Re^n}$ is the vector field which determines the continuous evolution of the system. For example, in location l, $\dot{x} \in F(l,x)$ determines continuous state trajectory of the system.
- $Inv : V \rightarrow 2^{\Re^n}$ determines the invariant set for each location. In each location l, the continuous state of the system cannot leave Inv(l).
- Jump : $V \times X \rightarrow V \times X$ determines the discrete evolution of the system.

The state of the system is in the form of (l,x), which l and x are discrete state and continuous state respectively. Therefore, this representation of the system, provides a comprehensive framework, which is able to consider both discrete and continuous part of the system, simultaneously.

A hybrid automaton can be represented by a graph, in which vertices V and the edge set E refer to the finite locations and discrete transitions, respectively (Fig.2). More precisely:

$$E = \{(l,l') \in V \times V \mid \ ((l,x),(l',x')) \in Jump\}$$

In this form of representation for each vertex $l \in V$ we have:

$$Init(l) = \{x \mid (l,x) \in Init\}$$

$$Inv(l) = \{x \mid (l,x) \in Inv\}$$

and for each edge $e = (l,l') \in E$ we associated a *Guard* set and a *Reset* map:

$$\begin{aligned} Guard(e) &= \{x \in Inv(l) \mid \exists\ x' \in Inv(l')\ s.t. \\ &\quad ((l,x),(l',x')) \in Jump\ \} \\ Reset(e) &= \{x' \in\ Inv(l')\ \mid \exists\ x \in Inv(l)\ s.t. \\ &\quad ((l,x),(l',x')) \in Jump\} \end{aligned}$$

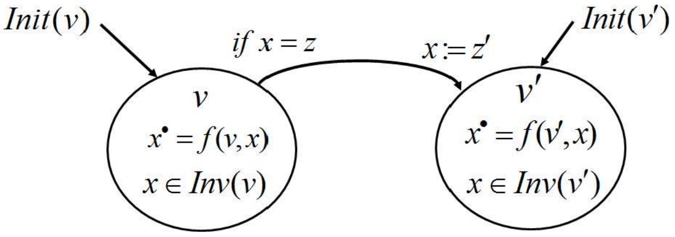


Fig. 2. Graph representation of a hybrid automaton.

The trajectory execution of a hybrid system is consisted of continuous evolution and discrete jumps, in which the continuous evolution of the system is determined by the vector field F and the discrete transitions are determined by the Jump relation. More precisely, any trajectory of the hybrid system starts from an initial state (l,x) $\in Init$ and the continuous part will be followed according to $\dot{x} = F(l,x)$, as long as x belongs to the invariant set *Inv(l)*. Over the continuous evolution, if one of the *Guard* conditions is satisfied, (i.e., $x \in$ *Guard (e)* for some $e = (l,l')$), then a discrete transition becomes enabled. In this case, if *Reset (e)* $= \{x'\}$, the location of the system will jump to l' and the continuous state will reset to x'.
It can be seen that this formalism is able to describe the behavior of either pure discrete, pure continuous, or interacting mixed continuous and discrete dynamics. The issues that can be considered in the area of hybrid systems are stability of hybrid systems [9], [10], [11], modeling and analysis of hybrid systems [12], [13], supervisory control of hybrid systems [14], [15], optimal hybrid control [16], reachability, verification and model checking of the hybrid systems [17], abstraction and approximate abstraction of hybrid systems [18], [19], [20], symbolic control of hybrid systems [21], [22], multi agent systems in hybrid framework [23] and so on.

III. HYBRID MODELING AND CONTROL OF A SINGLE UAV

A UAV is a hybrid system, capable of accomplishing different complex tasks like Take off, Landing, Hovering, and Cruising. Traditionally, for each task, a particular model is considered and correspondingly an especial controller is designed. In this strategy, switching between controllers will be designed such that different possible missions be achievable. As an example, a typical mission can be the observation of a particular area. This mission can be divided to some tasks such as launching vehicle, going to the objective area, observing the area, returning to the origin, recovering the vehicle, and finally end of the mission (Fig.3). Each task should be accomplished by selecting a particular controller and meanwhile, some continuous and discrete conditions should be checked to decide whether the task should be continued or jumped to another task. This leads the system to switch between controllers due to maneuver change of the UAV. In [24] such a supervisory control has been implemented for an underwater glider and the design procedure can be followed for a UAV. In this paper, they have considered a controller with three layers: Mission layer, Task Layer, and Behavioral layer. Each layer is a discrete event system (DES) which has been modeled by an automaton. This strategy is based on the supervisory control of discrete event systems which was initiated by Ramadge and Wonham in 1987 [25] and by now, it is a mature theory [26], [27], [28]; However, using DES supervisory control approach, the designed controller of this glider, is still a pure discrete controller and can be improved by hybrid supervisory control.

Hybrid modeling of a UAV firstly, facilitate designer to consider discrete and continuous dynamics, simultaneously. Secondly, it provides a useful tool to extend the design to a multi-agent scheme in which each UAV has its hybrid model and the whole system will satisfy some global specifications

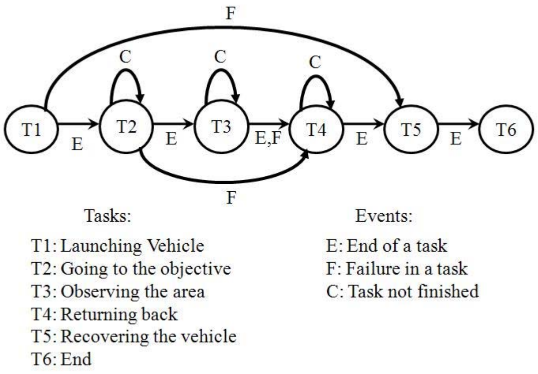


Fig. 3. Supervisor of the mission layer for observation an area

such as keeping formation, collision avoidance and obstacle avoidance. For instance, in [29] a hybrid controller for a single fixed wing UAV has been developed. This controller is composed of two separate and decoupled parts for the altitude and lateral control of the UAV. The lateral hybrid controller is rather messy and interested readers are referred to [29] for more details, however In Fig. 4, the hybrid controller for the altitude control part has been depicted. In this figure, G_{ij} are some *Guard* conditions that can be a user command, a sensor activation, or reaching to a particular state, and R_{ij} are some *Reset* values which the state of system will jump to these values whenever the *Guard* condition becomes satisfied. With these hybrid controllers for altitude and lateral dynamics of the system, a practical experiment has been done to accomplish an aerial surveillance mission.

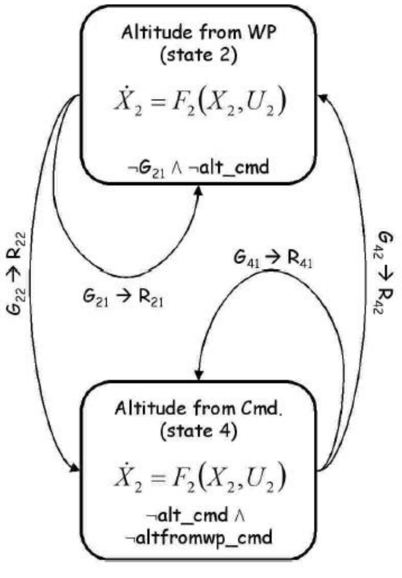


Fig. 4. A hybrid controller for altitude control of a single UAV.

Another approach to deal with a hybrid controller design problem is Mixed-Integer Linear Programming (MILP), which is able to convert a hybrid controller design problem into a smooth optimal control problem [30], [31], [32], [33].

For instance in [33], an optimal hybrid control problem of UAVs with logical constraints has been transferred to some inequality and equality constraints involving only continuous variables. Logical constraints are coming from symbolic specifications like obstacle avoidance, safety requirements, and collision avoidances or coming from physical constraints like maximum linear and angular accelerations.

The approach has then been applied to a mass-point model of a UAV and has been extended to multiple UAVs to solve an optimal path planning problem.

IV. FORMATION CONTROL OF UAVS IN HYBRID FRAMEWORK

Hybrid modeling of a single UAV, can be extended to a multi-agent cooperative system, using supervisory hybrid control scheme; However, only a few number of existing works have focused on the hybrid cooperative control of UAVs.

Formation control as a cooperative multi agent scheme, is the objective of most recent research projects in this area. In the formation control, it is important that UAVs keep a certain formation, while following a given trajectory. Moreover environment is dynamic and therefore, obstacle avoidance and collision avoidance should be embedded in the control design. More interesting and challenging problem is the reconfiguration of the flight formation of the UAVs. In reconfiguration problem, the formation of the UAV team, should be able to change, break down, and merge without collision and crash, to increase maneuverability and avoid dynamic obstacles. For instance, to avoid an obstacle, dividing the formation before reaching to the obstacle and again merging to reconstruct the original formation after the obstacle is a reconfiguration problem. The obstacle avoidance, therefore, can be handled more effectively by selecting such a dynamic flexible formation rather than a rigid formation. Furthermore, reconfiguration can be done according to the abilities of each member of UAV group. It means that in different missions like attack, defense, surveillance, and reconnaissance, a team of UAVs should take corresponding appropriate formation considering the abilities of the team members and requirements of that mission. Moreover, in most cases, a team of several simple UAVs are more reliable and cost effective than a complicated multi-task UAV. In addition, if we consider that damage, loss, and crash are common and unavoidable aerial accidents, UAVs should be able to take new formation in these cases as a safety campaign.

Generally, we can divide a formation problem into three parts: *Obtaining Formation*, *Keeping Formation*, and *Formation Reconfiguration*. Solving these problems, reveals the trajectory of each agent. These trajectories should be used as a setpoint for the lower level controllers of the UAVs to provide the corresponding control inputs. Recall that in all these cases, the constraints of the system should be satisfied and also the system should be driven in a safe region to avoid collision and obstacles. For instance, in [29], after developing a hybrid model of a single UAV, a formation control has been implemented for two actual UAVs. In this paper, they have implemented the hybrid controller such that the distance of all UAVs be greater than a minimum allowable distance to avoid collision. This can be expressed as:

$$\|P_i(t) - P_j(t)\| \geq \epsilon \ \forall t \geq 0, \forall_{i \neq j} \ i, j$$

where $P_i, \ i = 1, 2, ..., N$ is the position of the i'th UAV.
Furthermore, an input constraint could be the maximum linear or angular acceleration of each vehicle and can be expressed as:

$$\|a_i(t)\| \leq \epsilon \ \forall t \geq 0, \forall i$$

where a_i is the acceleration of i'th UAV.
The formation problem with these constraints for each agent can be considered as an example of an optimal hybrid control problem which is mentioned before.

In [34], the feasibility problem of formation has been addressed and in [35], the formation reconfiguration problem has been solved for a group of UAV including one actual UAV and three virtual UAVs. The addressed problem in this paper is as follows:
Given :

- *a group of autonomous vehicles*
- *an initial configuration*
- *a final configuration*
- *set of inter- and intra- vehicle constraints*
- *a time for configuration or reconfiguration*

determine a nominal state and input trajectory for each vehicle such that the group can start from the initial configuration and reach its final Configuration at the specified time while satisfying the constraints.
In that paper, four forms including line, diamond, wedge, and rectangle formation have been considered. The optimal formation reconfiguration problem to switch from each formation to another formation has been solved and the resulting trajectory for each UAV has been stored in its library. Then, according to the user commands, the formation is changed. To implement switching between two typical flight formations, two situations have been considered:

1) transient state: in which the UAVs are changing their formation from initial formation to reach final formation, but still it is not achieved.
2) Keeping the formation: in which the final formation already has been achieved and the controller should keep the achieved formation.

To implement this strategy, they have used a hybrid supervisory controller which meets all the specifications and then they have decomposed these hybrid controller to obtain a separate hybrid automaton for each UAV.

Another approach of implementing a hybrid controller has been used in [36]. A leader-follower decentralized hybrid control scheme with two fixed wing UAVs has been used for the simulation of the controller in a two dimensional space. The follower can fly either in left or right side of the leader with a fixed distance. Therefore, the follower is able to swap its position, particularly for avoiding collision and increasing the maneuverability. The hybrid controller is a decision maker for mode switching of the UAVs and is able to switch between tracking control, position swapping algorithm, and collision avoidance (Fig.5).

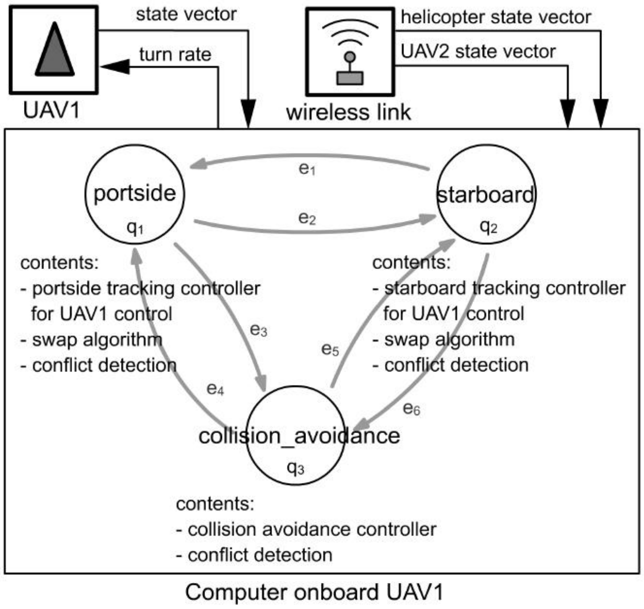


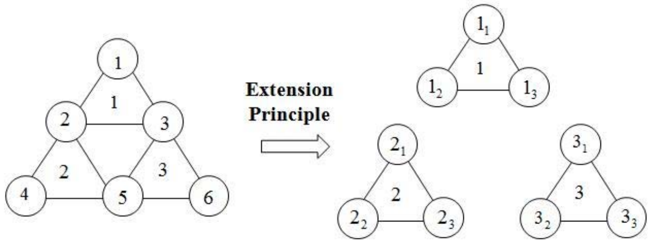
Fig. 5. Architecture of the hybrid controller of UAV1 as follower.

So far in all previous mentioned works, the formation algorithms were used for two or three agents. For the higher number of agents, the problem will be more complicated. In [37] firstly, using overlapping theorem [38], they have decomposed the graph of flight formation into some disjoint triangular subgraphs and they have obtained a control law for the formation control of each triangular subsystem. Then, again they have contracted these triangles to obtain the original graph. In fact, dealing with formation of triangles as a basic unit of flight formation is more rational than dealing with the formation of the whole graph. This is due to the characteristics of triangles. Since in a typical triangle, once lengths of all three edges are fixed, the triangle is determined.

The overlapping theorem gives some effective rules for extension and contraction of a graph into disjoint triangular subsystems. In Fig.6, extension of a flight formation graph including six agents into three triangular subsystems has been depicted. In this figure, agents 2, 3, and 5 are mutual nodes and belong to two triangular subsystems.

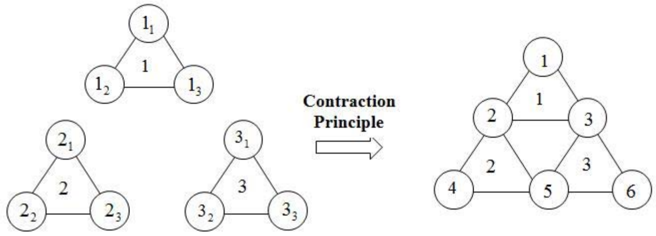
After obtaining control laws for each node, we should contract these subsystems to obtain the original formation flight, as it is shown in Fig.7.

In this strategy, for each subsystem, they have considered a leader and two follower. Consequently, a hybrid controller will be obtained for each leader and follower in order to keep the formation and avoid collision. Later, to follow the general path, a separate control signal will be added to all agents as a bias value, which will cause the center of the formation graph to follow the group path.



Original System	Expanded System
1	1_1
2	1_2 2_1
3	1_3 3_1
4	2_2
5	2_3 3_2
6	3_3

Fig. 6. Extension a graph into some disjoint triangular subsystems.



$$\begin{cases} u_1(t) = u_{1_1}(t) \\ u_2(t) = 1/2(u_{1_2}(t) + u_{2_1}(t)) \\ u_3(t) = 1/2(u_{1_3}(t) + u_{3_1}(t)) \\ u_4(t) = u_{2_2}(t) \\ u_5(t) = 1/2(u_{2_3}(t) + u_{3_2}(t)) \\ u_6(t) = u_{3_3}(t) \end{cases}$$

Fig. 7. Contraction of subsystems to obtain the original formation graph.

V. CONCLUSION

In this paper, we gave a brief review on the existing works in the area of hybrid modeling and control of UAVs in a cooperative scheme. Especially, we investigated the formation flight control in the hybrid framework. This study, shows that hybrid modeling and control of UAVs has offered a promising and competitive area and has emerged hot research demands with a great support of military and civilian applications.

Although hybrid modeling and control is an appropriate key for the problems of the aerial robotics, the application of hybrid theory for control of UAVs is not so mature. So far, most research works are ended with simulations and therefore, more practical experiments are required in future to support the simulation results. Furthermore, the problems like obstacle avoidance, collision avoidance, path generation, path tracking, and formation reconfiguration that conventionally have been solved via pure DES or by traditional pure continues approaches, should be reformulated in hybrid framework to utilize the advantage of hybrid modeling and analysis tools. Verification and model checking of the proposed models are necessary parts of future research projects for model validation. Moreover, after obtaining the hybrid model of UAVs, symbolic control of UAVs is future step to fulfill richer specifications.

In addition, in all these mentioned works, communication was assumed to be perfect, i.e., without failure and delay. it is necessary to investigate the effect of communication problems on the UAVs formation control. Furthermore, due to the lack of altitude control of the UAVs, most of the existing works on UAVs flight formation control, are implemented in a fixed height two dimensional horizontal space which are somehow similar to the existing results of formation control of the ground vehicles. Therefore, it will be beneficial to look for development of three dimensional flight formation algorithms and in parallel, it is necessary to make the low level controller of the vertical direction, more reliable to have more flexible maneuverability in three dimensional space and to reduce the disabilities of altitude controller of UAVs.

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